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KO‘NDALANG KESIMI IXTIYORIY STERJENLARNING HARORATNI HISOBGA OLGAN HOLDA DEFORMATSIYLANISH JARAYONLARINI SONLI TADQIQI

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Аннотация: *Maqolada haroratni hisobga olgan holda ko‘ndalang kesimi ixtiyoriy sterjenlarning kuchlanganlik-deformatsiyalanganlik holatini aniqlash uchun takomillashtirilgan samarali hisoblash algoritmi qaralgan. Ostrogradskiy-Gamiltonning umumlashgan variatsion tamoyili asosida haroratni hisobga olgan holda fazoviy yuklanishlardagi sterjenlarning kuchlanganlik-deformatsiyalanganlik holatini matematik modeli va hisoblash algoritmi ishlab chiqilgan. Sterjenlardagi tebranish jarayonlarining tenglamalari sistemasi, tabiiy boshlang‘ich va chegaraviy shartlari bilan ishlab chiqilgan. Markaziy chekli ayirmalar va matritsali haydash usullari asosida haroratni hisobga olgan holda fazoviy yuklanishlardagi sterjenlarning tebranishini statik va dinamik holatini hisoblash algoritmi ishlab chiqilgan. Natijalar grafik ko‘rinishda taqdim etilgan.*

Аннотация: *В статье рассмотрен усовершенствованный эффективный вычислительный алгоритм для определения напряжённо-деформированного состояния стержней произвольного поперечного сечения с учётом температурного воздействия. На основе обобщённого вариационного принципа Остроградского–Гамильтона разработаны математическая модель и вычислительный алгоритм для определения напряжённо-деформированного состояния стержней при пространственных нагрузках с учётом температуры. Сформирована система уравнений колебаний стержней с естественными начальными и граничными условиями. На основе центральных конечных разностей и матричного метода прогонки разработан алгоритм расчёта статического и динамического состояния колебаний стержней при пространственных нагрузках с учётом температурного воздействия. Результаты представлены в графическом виде.*

Abstract: *The article presents an improved and efficient computational algorithm for determining the stress–strain state of rods with arbitrary cross-sections considering temperature effects. Based on the generalized Ostrogradsky–Hamilton variational principle, a mathematical model and a computational algorithm have been developed to determine the stress–strain state of rods under spatial loads with temperature taken into account. A system of equations describing the vibration processes in the rods has been formulated together with natural initial and boundary conditions. Using central finite differences and the matrix sweep (Thomas) method, an algorithm has been developed to compute the static and dynamic vibration states of rods subjected to spatial loads considering temperature effects. The results are presented in graphical form.*

Kalit so‘zlar: *matematik model, sterjen, ixtiyoriy geometrik shakl, fazoviy yuklangan, tebranishlar, Gamilton–Ostrogradskiy variatsion prinsipi, kinetik va potensial energiya, harorat.*

Ключевые слова: *математическая модель, стержень, произвольная геометрическая форма, пространическая нагрузка, колебания, вариационный принцип Гамильтона–Остроградского, кинетическая и потенциальная энергия, температура.*

Keywords. *mathematical model, rod, arbitrary geometric shape, spatially loaded, oscillations, Hamilton-Ostrogradsky variational principle, kinetic and potential energy, temperature.*

Kirish. Jahonda fazoviy yuklanishlardagi sterjenlarning kuchlanganlik-deformatsiyalanganlik holatini geometrik nochiziqli masalalarini haroratni hisobga olgan holda

hal etishning matematik modellarini qurish, hisoblash algoritmini ishlab chiqish, sterjenlardagi deformatsion jarayonlarni baholash uchun avtomatlashtirilgan tizimlarni yaratishga yo‘naltirilgan ilmiy tadqiqotlar olib borilmoqda. Shu bilan birga ko‘ndalang kesimi ixtiyoriy murakkab shakldagi fazoviy yuklanishlardagi sterjenlarning chiziqli va geometrik nochiziqli masalalarni yechishning haroratni hisobga olgan holda matematik modellarini va algoritmlarini ishlab chiqishni ilmiy asoslash zarur hisoblanadi.

Respublikamizda sterjen tipidagi konstruksiyalarni nazariy asoslarini takomillashtirish va hisoblash usullarini ishlab chiqish bo‘yicha bir qator olimlar ilmiy tadqiqot ishlarini olib borishgan, jumladan akademik V.Q.Qobulov tomonidan konstruksiya elementlarining chiziqli va geometrik nochiziqli deformatsiyalanish jarayonlarini Ostrogradskiy-Gamilton tamoyili asosida aniqlashtirilgan nazariyasi ishlab chiqilgan va amaliy masalalarni yechishga algoritmik yondashuvlar taklif etilgan [1]. Keyinchalik akademik T.Bo‘riev, F.B.Badalov[2], B.Kurmanbaev[3], K.Sh.Bobomurodov, , T.Yuldashev[4], Sh.A.Nazirov[5], X.Eshmatov, B.Mardonov, A.A.Xaldjigitov[6], A.Abdusattarov[7], R.Sh.Indiaminov[8] Sh.A.Anarova[9], F.M.Nuraliev[10] va ularning shogirdlari tomonidan rivojlantirilmoqda. Sohaga oid tadqiqotlar tahlili shuni ko‘rsatadiki, fazoviy yuklanishlardagi ko‘ndalang kesimi ixtiyoriy sterjenlarning statik va dinamik, chiziqli va geometrik nochiziqli kuchlanganlik- deformatsiyalanganlik jarayonlarini haroatni hisobga olgan holda matematik modellashtirish hamda hisoblashning avtomatlashtirilgan tizimlarini takomillashtirish muammolari hozirgi kunda ham o‘rganilmoqda va rivojlantirilmoqda.

Masalaning qo‘yilishi. Bo‘ylama, ko‘ndalang va burovchi kuchlarning birgalikdagi ta’sirida fazoviy yuklanishlardagi sterjenlar nuqtalarining ko‘chish masalalarini matematik modellarini Ostrogradskiy-Gamiltonning umumlashgan variatsion tamoyili asosida ishlab chiqiladi [1, 12-17]:

$$\delta\int\left(K-\Pi+A\right)=0\,. \tag{1}$$

Matematik modellarni ishlab chiqishda Koshi geometrik munosabatlari, Guk qonunining teskari shakli va to‘g‘ri chiziqli koordinatalar tizimi ishlatiladi. Bu yerda K – kinetik energiya, Π^T -potensial energiya va A - tashqi kuchlar bajargan ish va ularning umumiy ko‘rinishi quyidagicha:

$$\begin{aligned} &K=\frac{1}{2}\rho\int_V\delta\sum_{i=1}^{3}\left[\frac{\partial^2u_i}{\partial t^2}u_i\right]dV,\ \Pi^T=\int_V\sum_{i=1}^{3}\sum_{j=1}^{3}\sigma_{ij}^T\delta\varepsilon_{ij}dVdt,\\ &\int_t\delta Adt=\int_V\sum_{i=1}^{3}F_i\,\delta u_i\,dV+\int_s\sum_{i=1}^{3}q_i\,\delta u_i\,ds+\int_{s_1}\sum_{i=1}^{3}f_i\,\delta u_i\,ds_1. \end{aligned} \tag{2}$$

Masalaning matematik modeli. Elastik deformatsiya va Vlasov-Djanelidze-Qobulovlarning aniqlashtirilgan nazariyalar asosida bo‘ylama, ko‘ndalang va burovchi kuchlarning birgalikdagi ta’sirini hisobga olgan holda fazoviy yuklanishlardagi sterjen nuqtalarining ko‘chishini quyidagi tengliklar ko‘rinishida ifodalash mumkin [1,11-17]:

$$\left.\begin{aligned} &u_1(x,y,z,t)=U(x,t)-z\alpha_1(x,t)-y\alpha_2(x,t)+\varphi(y,z)\nu(x,t)+\\ &+a_1\beta_1(x,t)+a_2\beta_2(x,t),\\ &u_2(x,y,z,t)=V(x,t)+z\theta(x,t),\\ &u_3(x,y,z,t)=W(x,t)-y\theta(x,t). \end{aligned}\right\} \tag{3}$$

bu yerda u_1, u_2, u_3 – ko‘chish vektorlari, U, V, W – sterjen markaziy o‘qining ko‘chishi, α_1, α_2 – sof egilishda ko‘ndalang kesimni markaziy o‘qqa nisbatan og‘ish burchagi, $\varphi(y,z)$ – Sen-Venanning buralish funksiyasi, β_1, β_2 - ko‘ndalang siljish burchagi, a_1, a_2 – berilgan funksiyalar θ - buralish burchagi, $\nu(x,t)$ - sterjen uzunligi bo‘yicha buralish burchagi.

Koshi munosabatlariga ko‘ra va (2) formulani hisobga olib fazoviy yuklanishlardagi deformatsiya komponentalari quyidagi ko‘rinishda aniqlanadi [1,14,16]:

$$\varepsilon_{ij} = \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\cdot\frac{\partial u_k}{\partial x_j}\right). \tag{4}$$

Kinetik energiya va tashqi kuchlar bajargan ishiga (3) ni qo‘yib, (2) dagi potensial energiyaga (4) ni qo‘yib natijaga (3) ni tadbiq etib, hamda variatsiya tenglamasi (1) muvozanat tenglamasiga qo‘yish natijasida sterjenning haroratni hisobga olgan holda tebranish tenglamasi kelib chiqadi [1,11-14, 16].

Hisoblash algoritmi. Fazoviy yuklanishlardagi sterjenlarning tebranishlari tenglamalar sistemalari skalyar holatda va o‘lchovlarda olinadi. Ostrogradskiy-Gamiltonning umumlashgan variatsion tamoyili asosida ishlab chiqishdan hosil bo‘lgan tenglamalar sistemasini analitik yechish murakkab bo‘lganligi uchun ifolarni sonli usullar orqali yechimini aniqlash uchun o‘lchovsiz holga keltiriladi [11-13].

Sterjen tebranish tenglamani sonli yechish uchun o‘zgaruvchilarni o‘lchovsiz ko‘rinishi quyidagicha:

$$W = a\bar{W},\ U = a\bar{U},\ V = a\bar{V},\ x = l\bar{x},\ y = b_0\bar{y},\ z = h\bar{z},\ v = \frac{1}{l}\bar{v},\ t = t_0\bar{t},\ \text{vaqt}\ t_0.\ t_0 = l\sqrt{\frac{\rho}{E}}.$$

munosabatlar bo‘yicha o‘lchovsiz miqdorlarni kiritib, cterjenning tebranish tenglamasi, boshlang‘ich, chegaraviy shartlarni $EF(a^2/l^2)$ ga bo‘lib yuboriladi hamda koeffitsientlardan tuzilgan matritsalar, vektorlar orqali tenglamani hosil qilamiz.

Sterjenlar tebranishining tenglamalar sistemasi, boshlang‘ich va chegaraviy shartlarining vektor-matritsa ko‘rinishi quyidagicha bo‘ladi:

$$M\frac{\partial^2\vec{U}}{\partial\bar{t}^{\,2}} + A\frac{\partial^2\vec{U}}{\partial\bar{x}^2} + B\frac{\partial\vec{U}}{\partial\bar{x}} + C\vec{U} + \left(\vec{\Phi}\frac{\partial\vec{U}}{\partial\bar{x}}\right)\frac{\partial^2\vec{U}}{\partial\bar{x}^2} + D\vec{\bar{F}}_i = 0, \tag{5}$$

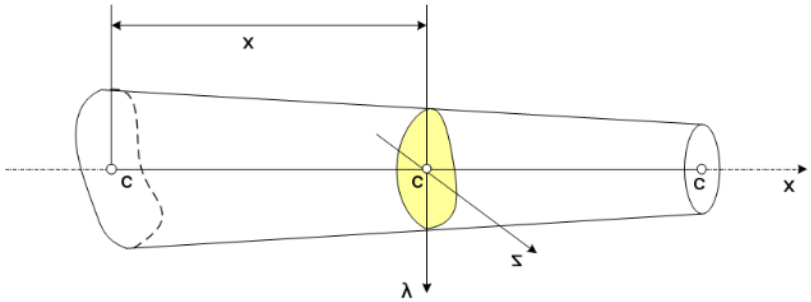
$$\bar{M}\left[\frac{\partial\vec{U}}{\partial\bar{t}}t_0\right]\delta\vec{U}\Big|_{\bar{t}} = 0, \tag{6}$$

$$\bar{B}\frac{\partial\vec{U}}{\partial\bar{x}} + \bar{C}\vec{U} + \left(\vec{\bar{\Phi}}\frac{\partial\vec{U}}{\partial\bar{x}}\right)\frac{\partial\vec{U}}{\partial\bar{x}} + \bar{D}\vec{\bar{F}}_{che} = 0, \tag{7}$$

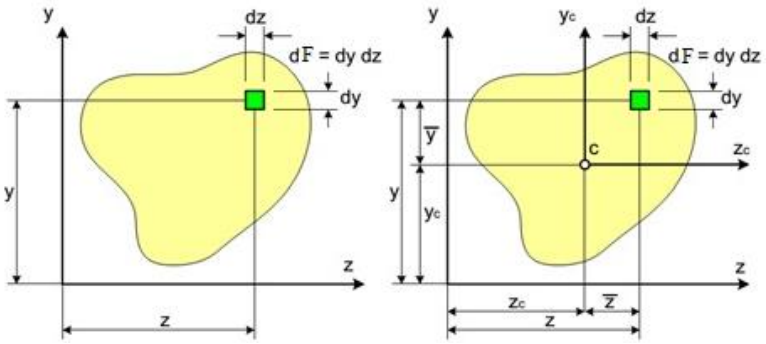
bu yerda: M, A, B, C, D, $\bar{M}$, $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ va $\vec{\Phi}$ matritsalar, $\vec{U}$, $\vec{\bar{F}}_i$, $\vec{\bar{F}}_{che}$ - tashqi kuchlar bajargan ishining vektorlar.

Yuqoridagi matematik model (5)-(7) daga matritsalar elementlarini sonli natijalarini olishda sterjen va uning geometrik xarakteristikasini hisoblash zarur.

Fazoviy yuklanishlardagi sterjenning cho‘zilishi va siqilishi - ko‘ndalang kesim yuziga, egilishi esa inersiya motenti, statik momenti, markazdan qochuvchi momenti, qutbli momenti qiymatlariga bog‘liq bo‘ladi (1-, 2-rasmlar).



1-rasm. Ko‘ndalang kesimi ixtiyoriy



2-rasm. Ko‘ndalang kesimi ixtiyoriy bo‘lgan sterjenning og‘irlik markazini ko‘rsatuvchi va kichik miqdordagi yuzacha

z_c, y_c - markaziy o‘qlari tizimiga o‘tishda dF maydonning elementar qismining koordinatalari quyidagicha o‘zgaradi (2-rasm):

$$\overline{y} = y - y_c, \qquad \overline{z} = z - z_c \ , \tag{8}$$

integrallarni hisoblash (8) ifodani hisobga olib, quyidagi formulalar bilan bajarilishi kerak bo‘ladi:

$$I_y = \int_y \int_z \overline{z}^2 dzdy,\ I_z = \int_y \int_z \overline{y}^2 dzdy,\ I_{zy} = \int_y \int_z \overline{zy} dzdy, \tag{9}$$

Ko‘ndalang kesimi ixtiyoriy sterjenning geometrik xarakteristikalari asosiy hisoblash algoritm sxemasi [17] da keltirib o‘tilgan.

Ko‘ndalang kesimi ixtiyoriy sterjenning mustahkamlik koeffitsienti elementlarini topishda kompyuter grafikasi imkoniyatlaridan foydalaniladi. Yuqoridagi algoritm asosida ko‘ndalang kesimi bo‘yicha geometrik parametrlarni hisoblab, so‘ng matritsa elementlariga hosil bo‘ladi hamda ko‘ndalang kesimi ixtiyoriy sterjenning tenglamalar sistemasi, boshlang‘ich shartlar va chegaraviy shartlar sonli natijalar olish mumkin bo‘ladi.

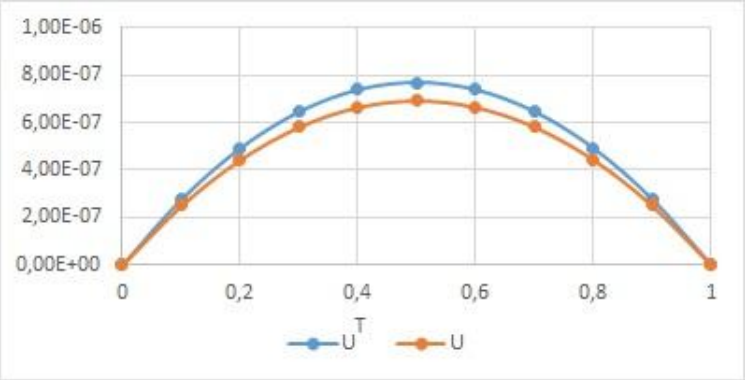
(5)-(7) ifodalarni chekli ayirmalar usulidan foydalanib tenglamani quyidagi ko‘rinishga keltiramiz [14-16].

$$A_1\vec{U}_{i+1}^{j+1} - B_1\vec{U}_i^{j+1} + C_1\vec{U}_{i-1}^{j+1} = \vec{\overline{F}}_i \tag{10}$$

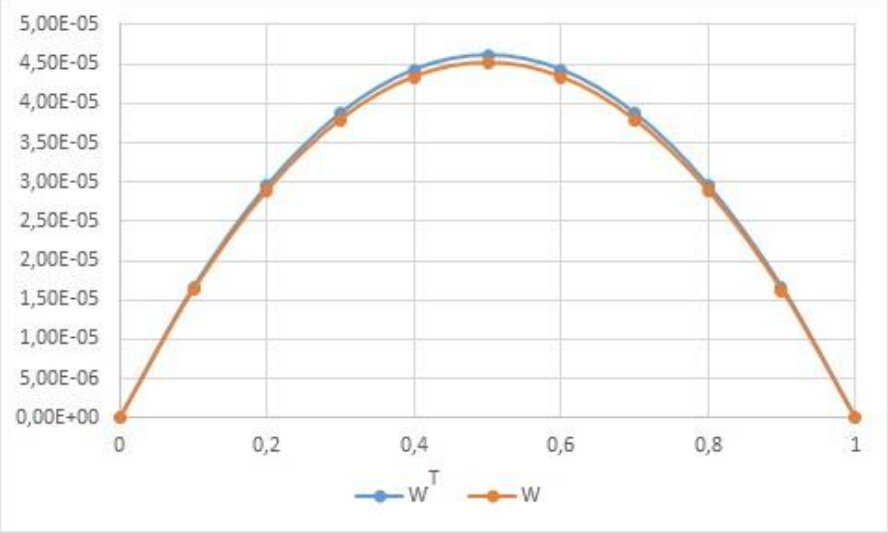
Hosil bo‘lgan (10) ifodani uch diagonalli tenglamalar sistemasini haydash usuli orqali yechim olinadi [15-16].

Hisoblash eksperimenti. Ikki tomoni maxkamlangan sterjenning haroratni hisobga olgan holda kuchlanganlik-deformatsiyalanganligini tekshiramiz. Sterjennnig geometrik va mexanik o‘lchamlari quyidagicha: Yung moluli $E = 2*10^5$, Puasson koeffitsienti $\mu = 0,3$, a=0.20 m, b=0.20 mm, l=2 m. Yuzalargi qo‘yilgan kuchlar qiymatlari $q_1 = 0,2\ H$, $q_2 = 0,1\ H$, $q_3 = 0,5\ H$, $M = 0,01\ Нм$.

Quyida olingan natijalar chegaraviy shartlarga mos keladi (3-, 4-rasmlar).

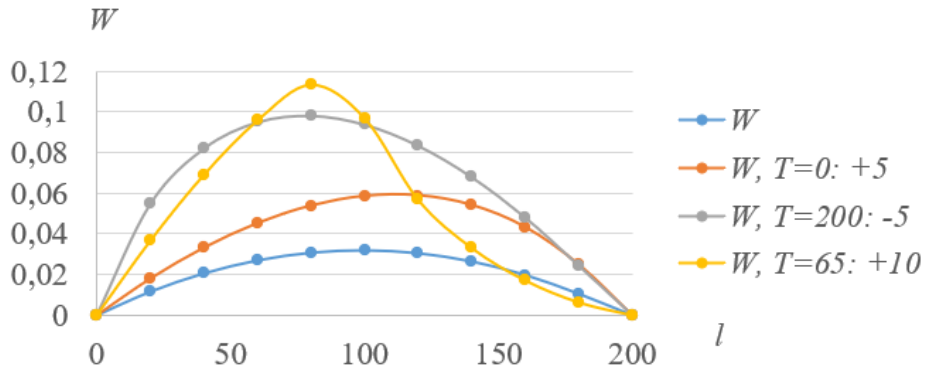


3-rasm. Haroratni hisobga olgan holda bo‘ylama tebranish qiyosiy grafigi



4-rasm. Haroratni hisobga olgan holda ko‘ndalang tebranish qiyosiy grafigi

Ikki tomoni mahkamlangan sterjenning haroratni hisobga olgan holda deformasiyalanganlik holatini tekshiramiz. Sterjenning geometrik o‘lchamlari *a*=10 cm, *b*=5 cm, *l*=200 sm, quyidagicha Yung moduli $E = 2*10^5\ МПа$, harorati - T=$50^0, 0^0$ ..+$5,200^0$..-5, 65^0..+10, Puasson koeffisiyenti - $\mu = 0,3$, Yuzalar orqali kuchlanganlik qiymatlari: $q_1 = 1\ H/см^2$, $q_2 = 1\ H/см^2$, $q_3 = 50\ H/см^2$.



5-rasm. Ko‘ndalang ko‘chish *W* ning sterjenь uzunligi bo‘yicha haroratining intensiv o‘zgarishidagi qiyosiy tahlili

Xulosa. Ostrogradskiy-Gamilton umumlashtirilgan variatsion tamoyiliga, elastik deformatsiya va Vlasov-Djanelidze-Qobulovlar tomonidan aniqlashtirilgan nazariyalarga asosan fazoviy yuklanishlardagi sterjenlarning chiziqli va geometrik nochiziqli masalalari uchun haaroratni hisobga olgan holda statikasi va dinamikasi uchun umumlashgan matematik model ishlab chiqildi. Ushbu model ixtiyoriy ko‘ndalang kesimli sterjenlarning bo‘ylama, ko‘ndalang va burovchi kuchlarning birgalikdagi ta’sirini hisobga olgan holda chiziqli va nochiziqli deformatsiyalanishi jarayonlarini to‘la ifodalashga xizmat qiladi.

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