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STABILITY ASSESSMENT OF HOMOGENEOUS WASTE DUMP SLOPES USING LIMIT EQUILIBRIUM METHODS

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Abstract. *External waste rock dumps formed with cyclical-and-continuous (cyclic-flow) mining technology at the Kalmakir open pit (Almalyk Mining and Metallurgical Complex, Uzbekistan) have reached heights of 85–94 m, and their slope stability governs both land use and industrial safety. This paper assesses the stability of four homogeneous dumps by four limit equilibrium methods that use a circular-cylindrical slip surface: the V.A. Florin method, the A.A. Nichiporovich method, the effective-stress method with A. Bishop's modification, and the G.L. Fisenko shear-resistance diagram method. The design strength of the dump material was obtained by reducing laboratory cohesion and friction by a safety factor of 1.2. For each dump the slip surface was divided into slices narrower than 1 m and the critical surface was found by searching a grid of centres. At an overall slope angle of 28° all four methods gave factors of safety of 1.33–1.34, differing by no more than 0.01. When the slope angle increases from 26° to 30°, the factor of safety falls from 1.43 to 1.25, about 0.045 per degree; the required value of 1.2 is reached near 31°. A check of the dump foundation by the maximum shear stress method gave a vertical stress of 1.90 MPa, a maximum shear stress of 0.60 MPa and a mean normal stress of 1.45 MPa under a 93 m dump. The results justify an overall slope angle of 27–28° for dumps of the present height and show that the simpler Fisenko method is adequate for homogeneous dumps on a rigid base.*

Annotatsiya. *Qalmoqqir ochiq konida (Olmaliq kon-metallurgiya kombinati, O‘zbekiston) siklli-oqimli texnologiya (SOT) yordamida shakllantirilgan tashqi tog‘ jinsi ag‘darmalari (otvallari) balandligi 85–94 metrga yetdi va ularning yonbag‘ir turg‘unligi yer resurslaridan samarali foydalanish hamda sanoat xavfsizligini ta’minlashda hal qiluvchi omil hisoblanadi. Ushbu maqolada doiraviy-silindrik sirpanish sirtidan foydalanuvchi to‘rtta chegaraviy muvozanat usuli yordamida to‘rtta bir jinsli ag‘darmaning turg‘unligi baholandi: V.A. Florin usuli, A.A. Nichiporovich usuli, A. Bishop modifikatsiyasidagi samarali kuchlanishlar usuli hamda G.L. Fisenkoning siljishga qarshilik diagrammasi usuli. Ag‘darma materialining hisobiy mustahkamligi laboratoriya sharoitidagi ilashish kuchi va ishqalanish ko‘rsatkichlarini 1,2 zaxira koeffitsiyentiga kamaytirish orqali aniqlandi. Har bir ag‘darma uchun sirpanish sirti 1 metrdan torroq bo‘laklarga (elementar ustunchalarga) ajratildi va markazlar to‘ri bo‘yicha qidirish orqali eng xavfli (kristik) sirt topildi. Yonbag‘irning umumiy qiyalik burchagi* 28° *bo‘lganda, barcha to‘rtta usul 1,33–1,34 oralig‘idagi turg‘unlik zaxirasi koeffitsiyentini ko‘rsatdi va ularning o‘zaro farqi 0,01 dan oshmadi. Qiyalik burchagi* 26° *dan* 30° *gacha ortganda, turg‘unlik zaxirasi koeffitsiyenti har bir daraja uchun taxminan 0,045 ga kamayib, 1,43 dan 1,25 gacha pasayadi; talab etiladigan 1,2 qiymati esa taxminan* 31° *atrofida qayd etiladi. Ag‘darma zaminining maksimal urinma kuchlanishlar usuli bo‘yicha tekshiruvi 93 metr balandlikdagi ag‘darma ostida vertikal kuchlanish 1,90 MPa, maksimal urinma kuchlanish 0,60 MPa va o‘rtacha normal kuchlanish 1,45 MPa ekanligini ko‘rsatdi. Olingan natijalar hozirgi balandlikdagi ag‘darmalar uchun* $27-28^\circ$ *umumiy qiyalik burchagini asoslaydi hamda qattiq zaminga joylashgan bir jinsli ag‘darmalar uchun soddaroq hisoblangan Fisenko usulining to‘liq yetarli ekanligini tasdiqlaydi.*

Аннотация. *Внешние отвалы вскрышных пород, сформированные с применением циклично-поточной технологии (ЦПТ) на карьере Кальмакыр (Алмалыкский горно-металлургический комбинат, Узбекистан), достигли высоты 85–94 м, и устойчивость их откосов определяет как эффективность землепользования, так и промышленную безопасность. В данной статье проведена оценка устойчивости четырех однородных отвалов четырьмя методами предельного равновесия, использующими*

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круглоцилиндрическую поверхность скольжения: методом В.А. Флорина, методом А.А. Ничипоровича, методом эффективных напряжений в модификации А. Бишопа и методом эпюр сопротивления сдвигу Г.Л. Фисенко. Расчетная прочность материала отвала определялась путем снижения лабораторных значений сцепления и трения на коэффициент безопасности 1,2. Для каждого отвала поверхность скольжения разбивалась на расчетные блоки шириной менее 1 м, а критическая поверхность определялась поиском по сетке центров. При общем угле откоса 28° *все четыре метода показали коэффициенты запаса устойчивости 1,33–1,34 с расхождением не более 0,01. При увеличении угла откоса с* 26° *до* 30° *коэффициент запаса устойчивости снижается с 1,43 до 1,25 (примерно на 0,045 на каждый градус); нормативное значение 1,2 достигается приблизительно при* 31°. *Проверка основания отвала методом максимальных касательных напряжений показала, что под 93-метровым отвалом вертикальное напряжение составляет 1,90 МПа, максимальное касательное напряжение — 0,60 МПа, а среднее нормальное напряжение — 1,45 МПа. Результаты обосновывают общий угол откоса* 27 – 28° *для отвалов существующей высоты и показывают, что более простой метод Фисенко вполне адекватен для расчетов однородных отвалов на жестком основании.*

Keywords: *waste rock dump; homogeneous slope; slope stability; factor of safety; limit equilibrium; circular slip surface; Bishop method; open pit mining.*

Kalit so‘zlar: *tog‘ jinslari ag‘darmasi (otval); bir jinsli yonbag‘ir; yonbag‘ir turg‘unligi; turg‘unlik zaxirasi koeffitsiyenti; chegaraviy muvozanat; doiraviy sirpanish sirti; Bishop usuli; ochiq kon ishlari.*

Ключевые слова: *отвал вскрышных пород; однородный откос; устойчивость откоса; коэффициент запаса; предельное равновесие; круглоцилиндрическая поверхность скольжения; метод Бишопа; открытые горные работы.*

Introduction. The volume of solid minerals extracted by open-pit mining continues to grow worldwide, while mining and geological conditions are becoming more difficult. Ore grades decline, pit depths increase and the volume of overburden to be removed rises accordingly. High-capacity mining and haulage equipment now allows open pits to be developed to depths of 600–700 m, which in turn requires the placement of very large volumes of overburden in external dumps [1, 2].

Cyclical-and-continuous (cyclic-flow) technology, in which trucks deliver rock to in-pit crushers and conveyors carry it to spreaders on the dump, makes it possible to build dumps that are higher and steeper than truck-formed dumps. Higher dumps reduce the land area taken out of use, but they also raise the risk of slope deformation. Slope failures on dumps damage haulage roads and conveyors, interrupt production and endanger people and equipment [3, 4].

Most dumps formed from crushed overburden can be treated as homogeneous bodies whose strength is described by the Mohr–Coulomb parameters of the mixed-size rock mass. For such slopes, limit equilibrium methods based on a circular-cylindrical slip surface remain the standard engineering tool. The first method of this kind was proposed by K.E. Petterson in 1916 and later adopted by the Swedish Geotechnical Commission; W. Fellenius developed it into the ordinary method of slices [5]. Subsequent methods by K. Terzaghi, K. Krey, D.W. Taylor, A.W. Bishop and others differ in which of the three equations of static equilibrium they satisfy and in how they treat the forces between slices [6–8].

In the former Soviet mining school, the methods of V.A. Florin, A.A. Nichiporovich and G.L. Fisenko were developed specifically for dumps and pit slopes, including rock masses that are not fully consolidated and contain pore water [9–11]. Because these methods rest on different assumptions, the choice among them affects the calculated factor of safety and therefore the design slope angle and height.

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The aim of this study is to compare four limit equilibrium methods for the homogeneous external dumps of the Kalmakir open pit, to determine the factor of safety of their slopes at the present heights, and to quantify how the overall slope angle controls stability.

The study covers four external dumps of the Kalmakir open pit of Almalyk MMC JSC: Dump No. 2 (section I), the Central dump, Dump No. 2 (section II) and the Special dump. The dumps are 85–94 m high and consist of crushed overburden of granite, granosyenite and syenite-diorite. The size distribution of the dump rock on the slopes is 0–100 mm – 54%, 100–200 mm – 21%, 200–300 mm – 13% and 300–400 mm – 12%.

The dumps rest on alluvial and deluvial sandy and loamy soils 2–30 m thick, underlain by a gravel layer 1.5–10 m thick and by weathered granite. In some boreholes the gravel was water-saturated below 9–10 m. In the present analysis the foundation is treated as rigid, and its stress state is checked separately (Section 2.4).

The design strength parameters of the dump material were obtained from laboratory values by applying a safety factor n = 1.2, as required by the guidelines for dump design [11, 12]:

$$C_p = \frac{C_M}{n}, \qquad \varphi_p = \arctan\frac{\tan\varphi_M}{n}$$

where C_M and φ_M are the laboratory cohesion and angle of internal friction of the dump material. The resulting parameters are given in Table 1.

Table 1

Height and strength parameters of the dumps

Dump	Height H, m	Unit weight γ, kN/m³	Cohesion C_M, kPa	Friction angle φ_M	Design cohesion C_p, kPa	Design friction angle φ_p
No. 2 (I)	93	20.4	25.0	35°36′	20.8	30°49′
Central	93	20.5	24.9	35°39′	20.8	30°52′
No. 2 (II)	94	20.2	25.3	34°45′	21.1	30°02′
Special	85	22.4	43.3	32°06′	36.1	27°36′

Four methods were applied, all based on a circular-cylindrical slip surface divided into vertical slices.

Florin method. The rock mass is treated as not fully consolidated. Slice *i* of width b_i is acted on by its weight P_i, the interslice reactions E_1 and E_2 (Krey scheme), the normal and tangential forces on its base and the pore water force U_i. The factor of safety follows from the moment equation about the centre of the slip circle [9]:

$$\eta = \frac{\sum \dfrac{(P_i - U_i)\sin\varphi + c\, b_i \cos\varphi}{\cos(\alpha_i - \varphi)}}{\sum P_i \sin\alpha_i + \dfrac{E_1 h_1 - E_2 h_2}{R}}$$

where α_i is the inclination of the slice base, R is the radius of the slip circle, and h_1, h_2 are the lever arms of the interslice forces.

Nichiporovich method. The equilibrium of forces on each slice is considered, and the hydrostatic, seepage and pore pressure forces (P_w, P_f, P_u) reduce the normal force on the base [10]:

$$\eta = \frac{\sum \left\{ \tan\varphi \left[P_i \cos\alpha_i - \left(P_w + P_f + P_u\right)_i \right] + c\, l_i \right\}}{\sum P_i \sin\alpha_i}$$

where l_i is the length of the slip surface within slice *i*.

Effective-stress method (Bishop modification). Shear resistance and pore pressure u_i enter the slice equilibrium according to the Coulomb law in effective stresses [7]:

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$$\eta = \frac{\sum[c' l_i + (P_i \cos\alpha_i - u_i l_i)\tan\varphi']}{\sum P_i \sin\alpha_i}, \qquad \tau = \frac{1}{\eta}[c' + (\sigma_n - u)\tan\varphi']$$

where c′ and φ′ are the effective strength parameters and σ_n is the normal stress on the slip surface.

Fisenko method. The total shear resistance of the rock is used without separating friction and cohesion. Along the developed slip surface, ordinates equal to the acting shear stress τ_i and the shear resistance $\tau_{r,i}$ are plotted, with the normal stress taken as $\sigma_n = \gamma\, h_i \cos^2\alpha_i$. The areas under the two curves give the sums of resisting and driving forces [11]:

$$\eta = \frac{\sum \tau_{r,i}\, l_i}{\sum \tau_i\, l_i}$$

For each dump the slip surface was divided into slices less than 1 m wide. The factor of safety was calculated for a grid of slip-circle centres, isolines of equal factor of safety were drawn, and the minimum value was taken as the factor of safety of the slope. Calculations were made for the present overall slope angle α = 28° and, to assess sensitivity, for α = 26–30° in steps of 1°. The stability of the dump foundation was checked with the stress field obtained from the theory of a linearly deformable medium [13]. The maximum shear stress and the mean normal stress at each point are

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}, \qquad \sigma_n = \frac{1}{2}(\sigma_x + \sigma_y)$$

The sums of driving and resisting forces along a trial surface are found from the mean values on each segment l_i, and the factor of safety is their ratio:

$$\eta_y = \frac{\sum_{i=1}^{n} \tau_{n,i+1/2}\; l_i}{\sum_{i=1}^{n} \tau_{max,i+1/2}\; l_i}, \qquad \tau_{i+1/2} = \frac{\tau_i + \tau_{i+1}}{2}$$

At the present overall slope angle of 28°, the factor of safety of all four dumps lies between 1.33 and 1.34 regardless of the method (Table 2). The Florin method gives 1.34 for every dump; the Nichiporovich, Bishop-modified and Fisenko methods each give 1.33.

Table 2

Factor of safety along the critical slip surface at α = 28°

Dump	H, m	Florin	Nichiporovich	Bishop (modified)	Fisenko
No. 2 (I)	93	1.34	1.33	1.33	1.33
Central	93	1.34	1.33	1.33	1.33
No. 2 (II)	94	1.34	1.33	1.33	1.33
Special	85	1.34	1.33	1.33	1.33

The factor of safety decreases steadily as the slope steepens (Figure 1). Between 26° and 30° it falls from 1.43 to 1.25. A linear fit to the five values gives a gradient of −0.045 per degree, so the required value of 1.2 would be reached at about 31°. Over the practical range of 27–30° the factor of safety is 1.25–1.38.

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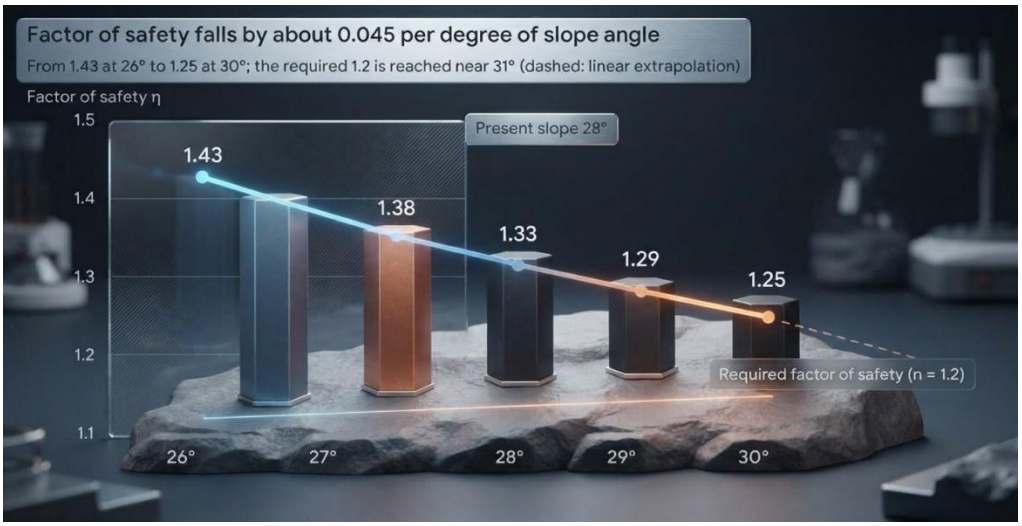

Fig. 1. Factor of safety of the Kalmakir dumps (H = 85–94 m) versus the overall slope angle

For the highest dump (H = 93 m, γ = 20.4 kN/m³) the vertical stress transmitted to the foundation is $\sigma_y = \gamma H \approx 1.90$ MPa. With a horizontal stress $\sigma_x = 1.00$ MPa and a shear stress $\tau_{xy} = 0.40$ MPa at the design point, the maximum shear stress is $\tau_{max} = 0.60$ MPa and the mean normal stress is $\sigma_n = 1.45$ MPa. The ratio $\tau_{max} / \sigma_n = 0.41$ corresponds to a mobilised friction angle of about 22.5°.

The four methods agree to within 0.01, although they rest on different assumptions about interslice forces and pore pressure. This agreement is expected for the conditions studied. The dumps are homogeneous, the dump material is coarse and free-draining, and the slip surface lies within the dump body above a rigid base. Under these conditions the pore pressure and seepage terms of the Florin, Nichiporovich and Bishop formulations are small, and all four expressions reduce to nearly the same ratio of resisting to driving forces. The slightly higher value of the Florin method (1.34) reflects its use of moment equilibrium with interslice reactions, which is known to give marginally higher values than force-equilibrium methods [6, 8].

The practical consequence is that, for homogeneous dumps on a rigid base, the simpler Fisenko method can be used for routine design without loss of accuracy. The more elaborate effective-stress methods remain necessary where the dump contains clayey, water-saturated material or rests on a saturated base, as is the case locally at Kalmakir where the gravel layer is saturated below 9–10 m.

A factor of safety of 1.33 at the present slope angle exceeds the required value of 1.2 by about 11%. The reserve, however, is consumed quickly as the slope steepens: each additional degree reduces the factor of safety by about 0.045, and at 30° only 4% of reserve remains. Since the actual slope angles of the external dumps at Kalmakir range up to 30–36° in some sections, local over-steepening is the main risk to stability. An overall slope angle of 27–28° is therefore recommended for dumps 85–94 m high, with steeper local angles permitted only on short-lived working slopes.

The foundation check indicates that the base, rather than the dump body, may control stability in some seasons. The mobilised friction angle of about 22.5° under a 93 m dump is close to the friction angle of the thawed foundation soils (up to 21° in summer) and below that of the frozen soils (25° in winter). This comparison is indicative only, because it uses the stress at a single point, but it shows that dumps on soft foundation soils require a separate check of the limiting height based on the bearing capacity of the base.

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The study has three limitations. The dump material was treated as homogeneous, whereas segregation during dumping produces a coarser toe and finer crest. Seismic loading was not considered. The foundation was assessed only at the design point, not along complete trial surfaces. These factors should be addressed in further work using numerical stress–strain modelling.

Conclusions

1. Four limit equilibrium methods with a circular-cylindrical slip surface (Florin, Nichiporovich, Bishop-modified effective stress and Fisenko) were applied to four homogeneous external dumps of the Kalmakir open pit, 85–94 m high.
2. At the present overall slope angle of 28° the factor of safety is 1.33–1.34 for all dumps and all methods, which exceeds the required value of 1.2 by about 11%.
3. The factor of safety decreases from 1.43 to 1.25 as the slope angle increases from 26° to 30°, at about 0.045 per degree; the required value is reached near 31°. An overall slope angle of 27–28° is recommended for dumps of the present height.
4. For homogeneous, free-draining dumps on a rigid base, the methods differ by no more than 0.01, so the simpler Fisenko method is adequate for routine design.
5. Under a 93 m dump the foundation carries a vertical stress of 1.90 MPa and a maximum shear stress of 0.60 MPa. The mobilised friction angle of about 22.5° is close to the strength of thawed foundation soils, so dumps on soft bases require a separate check of their limiting height.

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